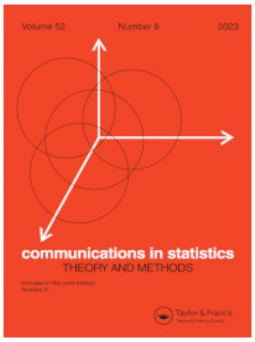

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A product acceptance decision-making method based on process capability with considering gauge measurement errors

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ABSTRACT
An acceptance sampling plan is an essential technique for quality assurance in manufacturing industries to help producers and buyers make appropriate decisions regarding many products. By providing the required sample sizes and critical value, the plan streamlines the quality standards process. The recent attention paid to acceptance sampling plans has tended to emphasize the process capability index while neglecting gauge measurement errors (GMEs), which have a direct impact on the fraction of defectives and decision-making processes to be the detriment of stakeholders. Thus, we provide the required sample size and the critical acceptance value considering GMEs. To demonstrate the impact of GMEs on the assessment of a product's lot, we present a real case study on a bilateral switch. Information on the required number of samples for the inspection and the acceptance critical value will help lead to a reliable decision.

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KEYWORDS
Acceptance sampling plan; fraction of defectives; process capability index; gauge measurement errors

1. Introduction

Producer and buyer satisfaction is closely related to product quality, and there are several methods to ensure that the product meets prescribed standards. One of them is to use an acceptance sampling plan that indicates the number of samples needed for inspection and the critical acceptance of values to make reliable decisions. To obtain these values, four parameters are required and each of them represents the interest of the producer and the buyer. The acceptable quality level (AQL), lot tolerance percent defectives (LPTD), producer's and buyer's risks, respectively. As Balamurali et al. (2020) point out, the combination of process capability indices (PCIs) in an acceptance sampling plan may reduce cost and offer more information about the product inspection for both parties (i.e. producers and buyers), and various sampling strategies for variable inspections have been developed. Pearn and Wu (2007) determined an effective decision-making method for product acceptance based on C_{pk} index, and they developed acceptance sampling determination for multiple characteristics based on C_{pk}^T index. Liu and Wu (2016) proposed a quick switching sampling system based on S_{pk} index.

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Aslam, Azam, and Jun (2013) proposed a mixed repetitive sampling plan based on C_{pk} index, and Aslam et al. (2013) considered multi-state repetitive group sampling plans for L_e index. More recently, Aslam, Balamurali, and Jun (2021) introduced a new multiple-dependent state sampling plan for C_{pk} index. These studies are representative of the acceptance sampling field using PCIs but do not consider gauge measurement errors (GMEs). GME is quantifying the measurement of a gauge repeatability and reproducibility. For example, Houf and Berman (1988) investigated thermal impedance performance measurement. A thermal impedance evaluator was used by three operators to measure the same samples three times. They showed that each operator has different measurement results, even when they did it in repetition. Thus, GME can occur due to instrument tests and influence by the operator. GMEs play an important role in the decision-making process. The reliability of this process depends on whether the GMEs are considered or not. Many authors showed that PCIs will be severely underestimated if we ignored the GMEs. For symmetric tolerances cases, Pearn and Liao (2005) evaluated C_{pk} index factoring for GMEs, and the results indicated that their omission renders power testing imperceptible. Wu (2011) applied the generalized confidence interval to investigate the impact of GMEs on the same index. For asymmetric tolerances cases Rakhmawati, Yang, and Wu (2016), Rakhmawati, Wu, and Yang (2016), Rakhmawati, Kim, and Sumiati (2020), and Grau (2011) showed the impact of GMEs on assessing PCIs. For one-sided tolerances, Grau (2013) showed the significant impact of GMEs to estimate the PCIs. Even for an incapability index, measurement errors might have a significant impact on detecting the process performance (Gildeh and Ganji, 2019). Most recently, Brik et al. (2019) assessed C^*_{p0} using a sampling plan in the presence of measurement system errors. Per our literature review, there is no available work on an acceptance sampling plan based on C_{pk} index accounts for GMEs even though C_{pk} is the most widely used index in manufacturing industries. To compensate for information needs regarding an acceptance sampling plan that accounts for GMEs, this paper investigates a C_{pk} index-based acceptance sampling plan in which the quality characteristics of the products follow a normal distribution.

2. Methods

2.1. Sampling distribution of C_{pk} index and its acceptance sampling plans

PCIs are extensively used in manufacturing industries to assess process capability. Each index with unique characteristics has been developed to represent the actual conditions in real-world applications. The C_p index proposed by Kane (1986) and known for its simplicity, is defined as

$$C_p = \frac{USL - LSL}{6\sigma} \tag{1}$$

where USL and LSL represent an upper and lower specification limit, respectively, and σ is the standard deviation of a process. The same author proposed the second index by taking into account for process departure from the midpoint of the specification limits M. It is defined as

$$C_{pk} = \frac{d - |\mu - M|}{3\sigma} \tag{2}$$

where $d = (USL - LSL)/2$, $M = (USL + LSL)/2$, and μ is the process mean. To estimate the natural estimator of C_{pk} index, Kane suggests using the following equation

$$\hat{C}_{pk} = \frac{d - |\bar{X} - M|}{3S} \tag{3}$$

where $\bar{X} = \sum_{i=1}^{n} X_i/n$ is the sample mean and $S = \sqrt{\sum_{i=1}^{n} (X_i - \bar{X})^2/(n-1)}$ is the sample standard deviation. Note that the above estimator is considered to be from a process with normal distribution and under statistical control. Pearn and Wu (2013) obtained an exact form of the cumulative distribution function (CDF) of the natural estimator $\hat{C}_{pk}$ using the integration techniques as follows:

$$F_{\hat{C}_{pk}}(x, b, \xi) = 1 - \int_0^{b\sqrt{n}} G\left(\frac{(n-1)\left(b\sqrt{n} - t\right)}{9nx^2}\right) \times \left(\phi\left[t + \xi\sqrt{n}\right] + \phi\left[t - \xi\sqrt{n}\right]\right) dt \tag{4}$$

for $x > 0$ and given $C_{pk} = C$, $b = d/\sigma$ can be expressed as $b = 3C + |\xi|$, $|\xi| = 3(C_p - C_{pk})$, $\xi = (\mu - M)/\sigma$, $\phi(\cdot)$ is the probability density function (PDF) of the standard normal distribution and $G(\cdot)$ is the CDF of the chi-square distribution χ^2_{n-1}. Thus, Equation (4) may be rewritten as

$$1 - F_{\hat{C}_{pk}}(x, b, \xi) = \int_0^{b\sqrt{n}} G\left(\frac{(n-1)\left(b\sqrt{n} - t\right)}{9nx^2}\right) \times \left(\phi\left[t + \xi\sqrt{n}\right] + \phi\left[t - \xi\sqrt{n}\right]\right) dt \tag{5}$$

To control the lot or process fraction defectives, a sampling plan is considered. Since the quality characteristic is variable, the acceptable values are defined according to its LSL and USL. Two points of the specified OC curve are used to design a sampling plan, $(AQL, 1 - \alpha)$ and $(LPTD, \beta)$.AQL and $LPTD$ can be described as levels of the product fraction of defectives corresponding to acceptable and unacceptable quality levels, respectively. The producer's risk, α and the consumer's risk, β values are commonly ranging from 0.01 to 0.10.

The testing hypothesis to determine whether a given process is capable or not is as follows

$$H_0 : p = AQL \text{ (process is capable)}$$

$$H_0 : p = LTPD \text{ (process is not capable)}$$

Hence, $H_0 : p =$ AQL is equivalent to $H_0 : C_{pk} = C_{AQL}$ which is to test process capability and is considered as the level of acceptable quality for C_{pk} index, corresponding to process a fraction of defectives AQL (in PPM).

Pearn and Wu (2013) explain the relationship between the index value and fraction defectives. Therefore, the required inspection sample size n and acceptance critical value c_0 for the sampling plan can be derived by finding the solutions of the following two nonlinear equations simultaneously:

$$\Pr\{\text{Accepting the product}|\text{fraction of defectives } p = AQL\ \} \geq 1-\alpha \tag{6}$$

$$\Pr\{\text{Accepting the product}|\text{fraction of defectives } p = LPTD\ \} \leq \beta \tag{7}$$

Given $C_{pk} = C,\ b$ can be expressed as $b = 3C + |\xi|$. Thus, the probability of accepting the product can be expressed as

$$\begin{aligned}\pi_{ASP}(C_{pk}) &= P\left(\hat{C}_{pk} \geq c_0|C_{pk}\right)\\ &= 1 - F_{\hat{C}_{pk}}(c_0, b, \xi)\end{aligned} \tag{8}$$

where $F_{\hat{C}_{pk}}(c_0, b, \xi) = 1 - \int_0^{b\sqrt{n}} G\left(\frac{(n-1)\left(b\sqrt{n}-t\right)}{9nc_0{}^2}\right) \times \left(\phi\left[t+\xi\sqrt{n}\right] + \phi\left[t-\xi\sqrt{n}\right]\right)dt,$ simply changed parameter x on Equation (4) by c_0. Accordingly, those nonlinear equations can be rewritten as

$$1-\alpha \leq 1 - F_{\hat{C}_{pk}}(c_0, b_1, \xi) \tag{9}$$

$$\beta \geq 1 - F_{\hat{C}_{pk}}(c_0, b_2, \xi) \tag{10}$$

where $b_1 = 3C_{AQL} + |\xi|$ and$b_2 = 3C_{LTPD} + |\xi|$. Pearn and Wu (2013) noted that the smallest possible value of n which is satisfied Equations (9) and (10) will be the required sample size n. They also suggested using $\xi = 1$ to obtain reliable n and c_0 without having to estimate the parameter ξ. Pearn and Liao (2005) also suggest the same value of ξ since the minimum lower bound obtains its minimum on that value for all cases. To solve Equations (9) and (10), they proposed the following equations:

$$S_1(n, c_0) \leq \left(1 - F_{\hat{C}_{pk}}(c_0, b_1, \xi)\right) - (1-\alpha) \tag{11}$$

$$S_2(n, c_0) \leq \left(1 - F_{\hat{C}_{pk}}(c_0, b_2, \xi)\right) - \beta \tag{12}$$

From Equations (11) and (12) they got the surface and contour plots for each equation and then plot them together to see the interaction. The solution to nonlinear simultaneous Equations (9) and (10) is the interaction of $S_1(n, c_0)$ and $S_2(n, c_0)$ in contour plots at level 0.

2.2. Sampling distribution of C_{pk} considering GMEs

The above section briefly discusses the sampling distribution of C_{pk} without considering GMEs. As we know, PCIs are unknown and estimated from the sample data. A certain amount of uncertainty can be present in the evaluation of the process performance and might lead to unreliable decisions. Therefore, providing accurate required sample size and decision-making rule for product sentencing in the presence of measurement errors is indeed necessary. Suppose that the relevant characteristic expressed as $X \approx N(\mu, \sigma)$, C_{pk} provides a measure of true capability. In practice, unfortunately, the observed variable $Y = X + G$ is examined rather than variable$X$ and G is measurement errors described as a random variable $G \approx N(0, \sigma_G^2)$. It can thus be assumed that$X$ and$G$ are stochastically independent so that $Y \approx N(\mu_Y = \mu, \sigma_Y^2 = \sigma^2 + \sigma_G^2)$. The PCI with contaminated data can be described as follows

$$C_{pk}^{Y}=\frac{d-|\mu_Y-M|}{3\sigma_Y} \tag{13}$$

For assessing process capability with contaminated data, Pearn and Liao (2005) defined the estimator of C_{pk} as

$$\hat{C}_{pk}^{Y}=\frac{d-|\bar{Y}-M|}{3S_Y} \tag{14}$$

where $\bar{Y}=\sum_{i=1}^{n}Y_i/n$ and $S_Y=\left[\sum_{i=1}^{n}(Y_i-\bar{Y})^2/(n-1)\right]^{1/2}$ are the estimators of mean and standard deviation for the empirical process, respectively. Pearn and Liao (2005) considered the sample data as the empirical data that contaminated with error, $Y_i, i=1,2,\ldots n$. To consider the GMEs on process capability assessment, the use of the gauge capability is needed. Montgomery and Runger (1993) defined it as

$$\lambda=\frac{6\sigma_G}{USL-LSL}\times 100\%. \tag{15}$$

Based on the preceding definitions, the relationship between the true and the empirical process capability can be defined as follows:

$$\frac{C_{pk}^{Y}}{C_{pk}}=\frac{1}{\sqrt{1+\lambda^2C_p^2}} \tag{16}$$

The CDF of $\hat{C}_{pk}^{Y}$ can thus be defined as follows:

$$F_{\hat{C}_{pk}^{Y}}(x)=1-\int_0^{b^Y\sqrt{n}} G\left(\frac{(n-1)\left(b^Y\sqrt{n}-t\right)}{9nx^2}\right)\times\left(\phi\left[t+\xi^Y\sqrt{n}\right]+\phi\left[t-\xi^Y\sqrt{n}\right]\right)dt \tag{17}$$

where $b^Y=3C_p^Y$, $\xi^Y=3(C_p^Y-C_{pk}^Y)$, $C_p^Y=C_p/\sqrt{1+\lambda^2C_p^2}$, $C_{pk}^Y=C_{pk}/\sqrt{1+\lambda^2C_p^2}$ (see Pearn and Liao, 2005).

2.3. Process yield of $\hat{C}_{pk}^{Y}$considering GMEs

According to Chang and Wu (2008), process yield is a part of measuring process performance, it is represented by the percentage of units passing the inspection. Under the normality assumption, Boyles (1991) provided the upper and the lower bounds on yield associated with C_{pk} as $2\Phi(3C_{pk})-1\leq \text{yield}\leq\Phi(3C_{pk})$, where $\Phi(x)$ is the CDF of the standard normal distribution. Furthermore, in terms of non-conforming PPM (part per million), we can calculate the process yield as follows

$$10^6\times\Phi(-3C_{pk})\leq PPM\leq 10^6\times 2\times\Phi(-3C_{pk}). \tag{18}$$

To investigate the behavior of PPM in consideration of GMEs, we substitute C_{pk} with C_{pk}^{Y} as follows

$$10^6\times\Phi\left(-3C_{pk}^{Y}\right)\leq PPM\leq 10^6\times 2\times\Phi\left(-3C_{pk}^{Y}\right). \tag{19}$$

Table 1. Index values and the corresponding nonconforming units (PPM) for $\lambda = 0.00, 0.05, 0.10, 0.15$.

	λ							
	0.00		0.05		0.10		0.15	
C_{pk}	Lower bound	Upper bound	Lower bound	Upper bound	Lower bound	Upper bound	Lower bound	Upper bound
0.60	35930	71861	36084	72168	36545	73090	37315	74629
0.65	25588	51176	25728	51455	26147	52294	26849	53698
0.70	17864	35729	17987	35974	18356	36712	18976	37952
0.75	12224	24449	12329	24658	12643	25287	13174	26348
0.80	8198	16395	8284	16567	8544	17087	8985	17969
0.85	5386	10772	5455	10910	5664	11327	6020	12040
0.90	3467	6934	3520	7041	3683	7367	3963	7926
0.95	2186	4372	2226	4453	2350	4700	2564	5128
1.00	1350	2700	1379	2759	1471	2941	1630	3261
1.05	816	1633	837	1675	903	1806	1019	2038
1.10	483	967	498	996	544	1088	626	1253
1.15	280	561	290	580	321	643	379	757
1.20	159	318	166	331	186	373	225	450
1.25	88	177	93	185	106	212	132	263
1.30	48	96	51	101	59	119	76	152
1.33	33	66	35	70	41	83	54	108
1.35	26	51	27	54	32	65	43	86
1.40	13	27	14	29	17	35	24	48
1.45	7	14	7	15	9	18	13	26
1.50	3	7	4	7	5	10	7	14
1.55	2	3	2	4	2	5	4	8
1.60	1	2	1	2	1	2	2	4
1.65	0	1	0	1	1	1	1	2
1.67	0	1	0	1	0	1	1	2
1.70	0	0	0	0	0	1	1	1
1.75	0	0	0	0	0	0	0	1
1.80	0	0	0	0	0	0	0	0
1.85	0	0	0	0	0	0	0	0
1.90	0	0	0	0	0	0	0	0
1.95	0	0	0	0	0	0	0	0
2.00	0	0	0	0	0	0	0	0

Tables 1 and 2 display various values of C_{pk} with a step of 0.05 between 1.00 and 2.00, and the corresponding possible lower and upper bounds of nonconforming units in PPM for different $\lambda = 0.00, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30$. We also set the index value of 1.33 and 1.67, which according to Pearn and Wu (2007) are important as minimum capability requirements in manufacturing industries.

Figures 1 and 2 demonstrate that the nonconforming units increase as λ increases. This phenomenon aligns with the theory that the true process capability would be underestimated if we calculate the process capability based on the contaminated data. Note that the chosen indices values in these are based on the commonly used parameter in manufacturing industries.

3. Results

3.1. Designing C_{pk} acceptance sampling plans considering GMEs

Most research works related to an acceptance sampling plan are carried out under the assumption of no GMEs. Unfortunately, such an assumption does not reflect the

Table 2. Index values and the corresponding nonconforming units (PPM) for $\lambda = 0.20, 0.25, 0.30$.

	λ					
	0.20		0.25		0.30	
C_{pk}	Lower bound	Upper bound	Lower bound	Upper bound	Lower bound	Upper bound
0.60	38393	76786	39781	79561	41478	82956
0.65	27836	55673	29113	58227	30685	61370
0.70	19852	39704	20993	41986	22407	44814
0.75	13929	27858	14920	29839	16159	32318
0.80	9617	19234	10455	20910	11515	23029
0.85	6536	13071	7227	14453	8112	16225
0.90	4373	8746	4929	9859	5654	11307
0.95	2882	5763	3320	6639	3900	7801
1.00	1871	3741	2208	4417	2665	5330
1.05	1197	2394	1452	2904	1805	3609
1.10	755	1510	944	1887	1212	2424
1.15	470	939	607	1214	808	1615
1.20	288	576	386	773	534	1069
1.25	175	349	244	487	351	702
1.30	104	209	152	304	230	459
1.33	76	153	114	228	177	355
1.35	62	123	94	188	149	298
1.40	36	72	58	116	97	193
1.45	21	42	35	71	62	124
1.50	12	24	21	43	40	80
1.55	7	13	13	26	26	51
1.60	4	8	8	15	16	33
1.65	2	3	4	7	9	17
1.67	2	4	5	9	10	21
1.70	1	2	3	5	7	13
1.75	1	1	1	3	4	8
1.80	0	1	1	2	3	5
1.85	0	0	1	1	2	3
1.90	0	0	0	1	1	2
1.95	0	0	0	0	1	1
2.00	0	0	0	0	0	1

practical situations when GMEs are inevitable. Thus, in this paper, we consider a sampling plan variable to control the process fraction of defectives by accounting for GMEs. The acceptance sampling plan for C_{pk} without considering GMEs can be applied by replacing some parameters for considering GMEs. By substituting b by b^Y, b_1 by $b_1^Y = \left(3\left(C_{AQL}/\sqrt{1+\lambda^2 C_p^2}\right) + |\xi_Y|\right)$, b_2 by $b_2^Y = \left(3\left(C_{LTPD}/\sqrt{1+\lambda^2 C_p^2}\right) + |\xi_Y|\right)$, ξ by ξ^Y to Equations (8)–(12), we can get Equations (20)–(22) as follows:

$$\pi_{ASP}^Y\left(C_{pk}^Y\right) = 1 - F_{\hat{C}_{pk}^Y}\left(c_0, b^Y, \xi_Y\right) \tag{20}$$

$$1-\alpha \le 1 - F_{\hat{C}_{pk}^Y}\left(c_0, b_1^Y, \xi_Y\right) \tag{21}$$

$$\beta \ge 1 - F_{\hat{C}_{pk}^Y}\left(c_0, b_2^Y, \xi_Y\right) \tag{22}$$

$$S_1^Y(n, c_0) \le \left(1 - F_{\hat{C}_{pk}^Y}\left(c_0, b_1^Y, \xi_Y\right)\right) - (1-\alpha) \qquad (21)$$

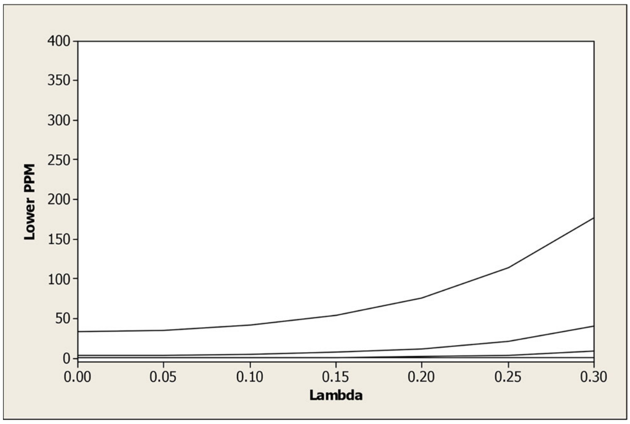

Figure 1. Lower bounds on nonconforming units in PPM versus λ for $C_{pk} = 1.33$, 1.15, 1.67, 2.00 (from top to bottom in plot).

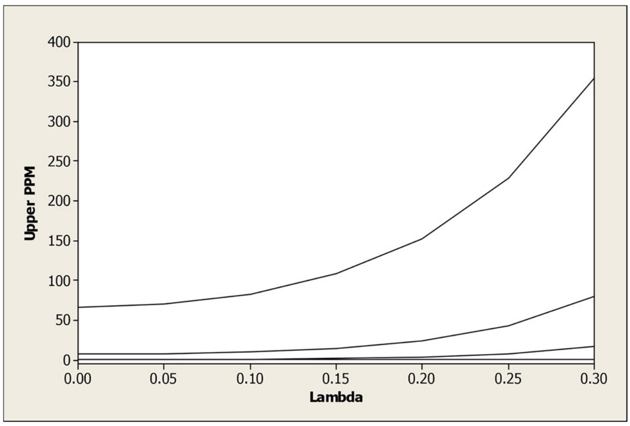

Figure 2. Upper bounds on nonconforming units in PPM versus λ for $C_{pk} = 1.33$, 1.15, 1.67, 2.00 (from top to bottom in plot).

$$S_2^Y(n, c_0) \le \left(1 - F_{\hat{C}_{pk}^Y}\left(c_0, b_2^Y, \xi_Y\right)\right) - \beta \tag{22}$$

For $C_{AQL} = 1.33$ and $C_{LTPD} = 1.00$, Figures 3 and 4 display the surfaces and contour plots of Equations (21) and (22) simultaneously for $\lambda = 0.10$, with $\alpha = 0.025$, and $\beta = 0.01$, respectively. Figures 9 and 10 demonstrate that the interaction between $S_1^Y(n, c_0)$ and $S_2^Y(n, c_0)$ occurs in contour curves at level 0 and $(n, c_0) = (137, 1.1630)$ for $\lambda = 0.10$, which is the solution of nonlinear Equations (21) and (22). In other words, the minimum required sample size $n = 137$ and critical acceptance value $c_0 = 1.1630$ of the sampling plan are based on the capability indexC_{pk}^Y. On the other hand, for$\lambda = 0.00$, (n, c_0) is equal to $(136, 1.1790)$ based on the capability index C_{pk} (please see Table 3). Thus, if the measurement is contaminated with errors and we compare it with a relevant acceptance of critical value, the product lots may be erroneously rejected.

For practical applications, we calculate and tabulate the required sample sizes (n) and the critical acceptance values (c_0) for the sampling plans considering GMEs. Those

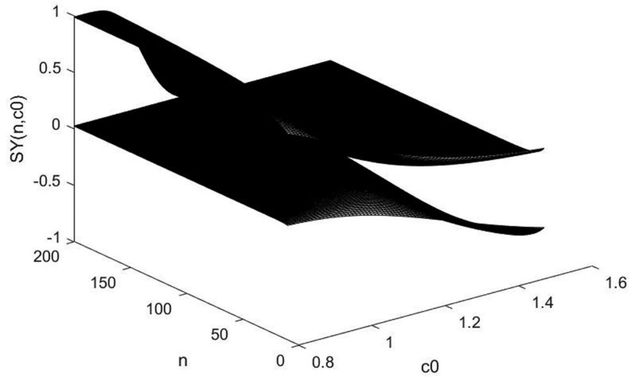

Figure 3. Surface Plot of S_1^Y and S_2^Y for $C_{AQL} = 1.33,\ C_{LTPD} = 1.00,\ \alpha = 0.025,\ \beta = 0.01,\ \lambda = 0.10$..

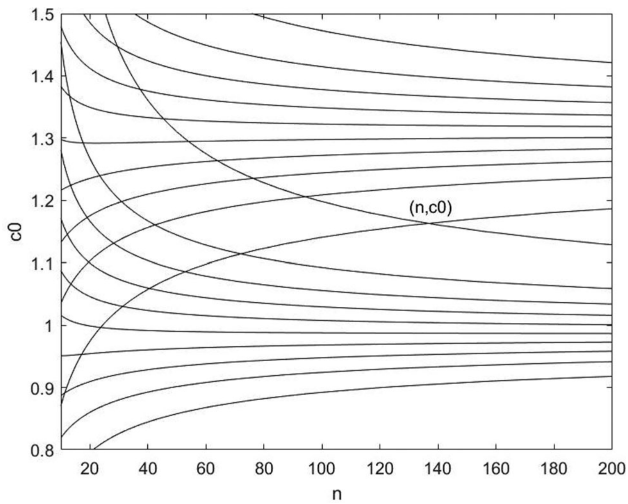

Figure 4. Contour Plot of S_1^Y andS_2^Y for $C_{AQL} = 1.33,\ C_{LTPD} = 1.00,\ \alpha = 0.025,\ \beta = 0.01,\ \lambda = 0.10$..

values for different combinations of producer's-risk and buyer's-risk with various benchmarking quality levels of (C_{AQL}, C_{LPTD}) and λ can be see on the Appendix. Several value of $\alpha = 0.010, 0.025, 0.050, 0.075, 0.100$ and $\beta = 0.010, 0.025, 0.050, 0.075, 0.100$ representing commonly used producer's and consumer's risk, respectively. Various commonly used benchmarking quality levels, $(C_{AQL}, C_{LPTD}) = (1.33,\ 1.00),\ (1.50,\ 1.33),\ (1.67,\ 1.33)$ and (2.00, 1.67) are representing the level of acceptability correspond to the lot or process fraction defectives AQL and LPTD, respectively. Several degrees of contamination of GME $\lambda = \{0.00,\ 0.05,\ 0.10,\ 0.15,\ 0.20,\ 0.25,\ 0.30\}$ representing small to large errors contamination are considered. Note that in practice, λ value can be obtained by conducting a gauge Repeatability and Reproducibility (R&R) study. It is used to define the amount of variation in the measurement data (i.e., equipment variation and operator variation) due to the measurement system. It then compares measurement variation to the total variability observed, thus the capability of the measurement system can be obtained. The user can calculate a sampling plan value that is not tabulated by solving Equations (21) and (22), then using

Table 3. Required sample sizes (n) and critical values (c_0) for various α and β with selected $C_{AQL} = 1.33$, $C_{LTPD} = 1.00$ and $\lambda = 0.00, 0.05, 0.10, 0.15, 0.20$.

		λ									
		0.00		0.05		0.10		0.15		0.20	
α	β	n	c_0	n	c_0	n	c_0	n	c_0	n	c_0
0.01	0.01	158	1.1645	158	1.1605	159	1.1487	159	1.1299	160	1.1050
	0.025	132	1.1510	132	1.1470	133	1.1354	133	1.1167	134	1.0921
	0.05	112	1.1372	112	1.1333	113	1.1218	113	1.1034	114	1.0791
	0.075	100	1.1271	100	1.1233	100	1.1119	101	1.0936	101	1.0695
	0.10	91	1.1186	91	1.1148	91	1.1035	92	1.0854	92	1.0614
0.025	0.01	136	1.1790	136	1.1750	137	1.1630	137	1.1440	138	1.1188
	0.025	113	1.1655	113	1.1615	113	1.1497	114	1.1308	114	1.1059
	0.05	94	1.1517	94	1.1477	94	1.1361	95	1.1175	96	1.0928
	0.075	83	1.1414	83	1.1375	83	1.1260	84	1.1075	84	1.0831
	0.10	75	1.1327	75	1.1288	75	1.1173	75	1.0990	76	1.0748
0.05	0.01	119	1.1937	119	1.1896	119	1.1776	120	1.1582	121	1.1327
	0.025	97	1.1805	97	1.1764	97	1.1645	98	1.1454	98	1.1202
	0.05	80	1.1669	80	1.1629	80	1.1511	81	1.1322	81	1.1072
	0.075	70	1.1566	70	1.1526	70	1.1409	70	1.1222	71	1.0974
	0.10	62	1.1477	62	1.1438	62	1.1322	63	1.1136	63	1.0890
0.075	0.01	108	1.2047	108	1.2006	109	1.1884	109	1.1689	110	1.1431
	0.025	87	1.1919	87	1.1878	88	1.1757	88	1.1564	89	1.1309
	0.05	71	1.1785	71	1.1745	71	1.1625	72	1.1435	72	1.1182
	0.075	62	1.1683	62	1.1643	62	1.1525	62	1.1336	62	1.1086
	0.10	55	1.1595	55	1.1555	55	1.1438	55	1.1250	55	1.1002
0.10	0.01	100	1.2140	101	1.2098	101	1.1976	101	1.1779	102	1.1520
	0.025	80	1.2016	80	1.1975	81	1.1853	81	1.1659	82	1.1402
	0.05	65	1.1886	65	1.1845	65	1.1725	65	1.1533	66	1.1278
	0.075	56	1.1787	56	1.1746	56	1.1627	56	1.1436	57	1.1184
	0.10	49	1.1700	49	1.1659	49	1.1541	50	1.1351	50	1.1101

the solving system of nonlinear equations in Matlab software. In this case, we used the "fsolve" command to solve the problem.

If the benchmarking quality level (C_{AQL}, C_{LPTD}) is set to$(1.33, 1.00)$ with $\alpha = 0.01$, $\beta = 0.05$, and $\lambda = 0.1$, then the corresponding sample size and critical acceptance value would be obtained as $(n, c_0) = (113, 1.1167)$. The lot will be accepted if the 113 inspected product items yield measurements with $\hat{C}_{pk}^{Y} \geq 1.1167$. Figures 5–12 demonstrate that n increases as λ increases (with an accuracy up to 10^{-3}) and c_0 decreases as λ increases. This means disregarding GMEs results in an underestimation of true process capability. To obtain a reliable decision we should thus use the adjusted required sample size and acceptance critical value.

Besides, the greater α, β, the smaller the sample size required for inspection, which can cause the customer to suffer from accepting a bad lot. Therefore, a larger sample size is required to reduce the number of defective products considered good. Furthermore, for fixed α, β, andC_{LTPD}, the required sample size decrease in inverse proportion to C_{AQL}.

On Tables 3–10, n and c_0 are obtained based on given values of α, β, AQL, and LPTD. If the estimated C_{pk}^{Y} value is greater than c_0, then the consumer accepts the product. Otherwise, we lack sufficient information to conclude that the process meets the present capability requirement, hence the consumer will reject the product lot. The procedure for the proposed sampling plan is as follows:

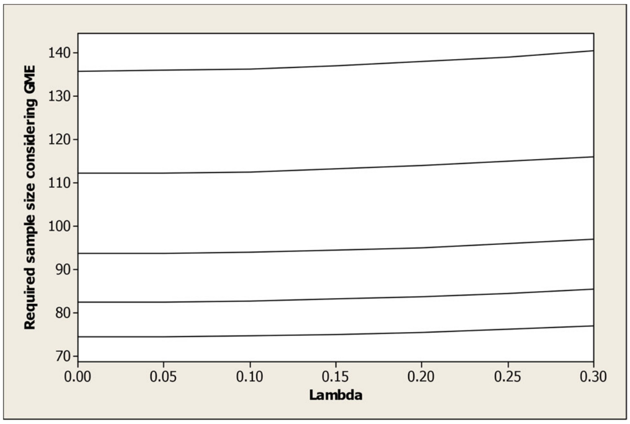

Figure 5. A plot of n with λ for $C_{AQL} = 1.33$, $C_{LPTD} = 1.00$, $\alpha = 0.025$, $\beta = 0.010$, 0.025, 0.050, 0.075, 0.100 (from top to bottom).

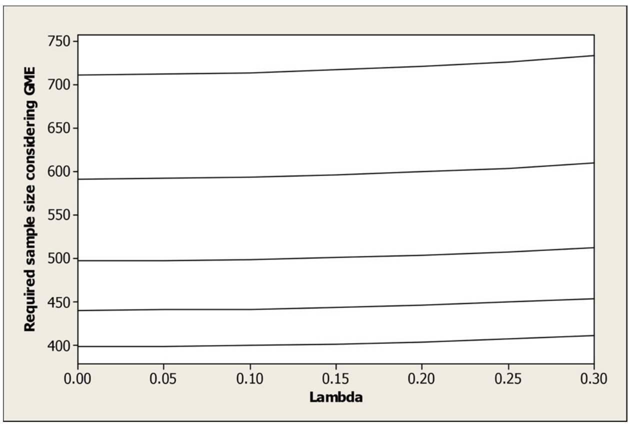

Figure 6. A plot of n with λ for $C_{AQL} = 1.55$, $C_{LPTD} = 1.00$, $\alpha = 0.025$, $\beta = 0.010$, 0.025, 0.050, 0.075, 0.100 (from top to bottom).

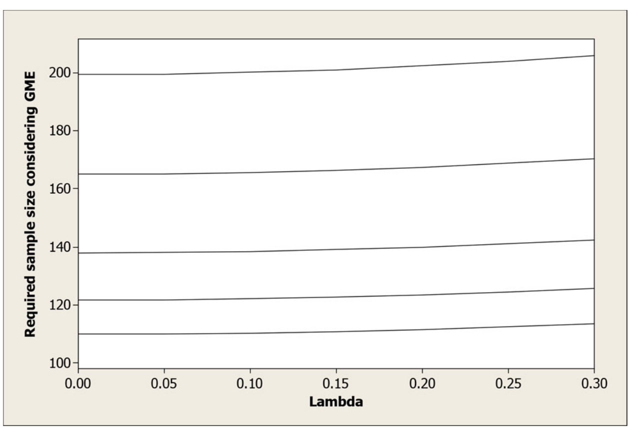

Figure 7. A plot of n with λ for $C_{AQL} = 1.67$, $C_{LPTD} = 1.33$, $\alpha = 0.025$, $\beta = 0.010$, 0.025, 0.050, 0.075, 0.100 (from top to bottom).

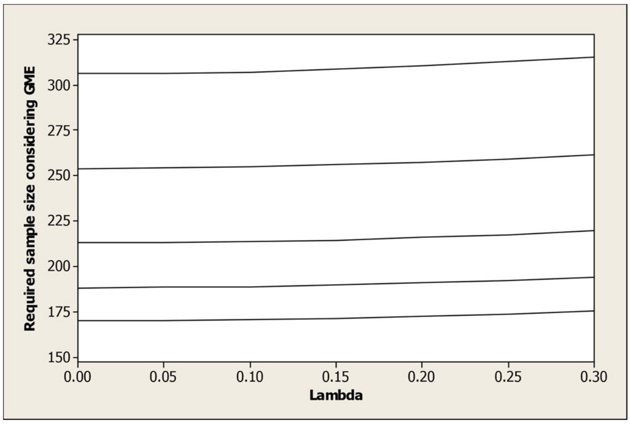

Figure 8. A plot of n with λ for $C_{AQL}= 2.00$, $C_{LPTD} = 1.67$, $\alpha= 0.025$, $\beta = 0.010, 0.025, 0.050, 0.075, 0.100$ (from top to bottom).

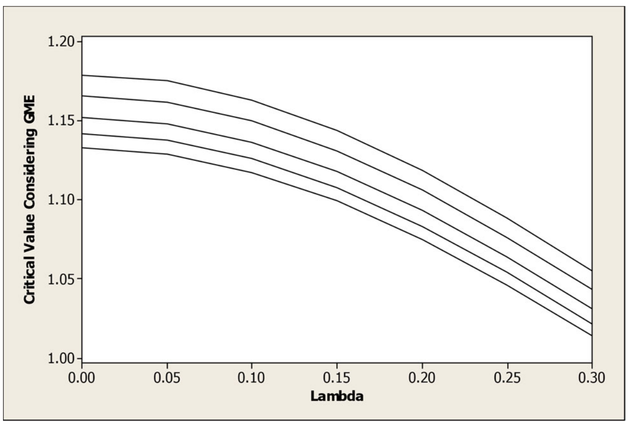

Figure 9. A plot of c_0 with λ for $C_{AQL} = 1.33$, $C_{LPTD} = 1.00$, $\alpha= 0.025$, $\beta = 0.010, 0.025, 0.050, 0.075, 0.100$ (from top to bottom).

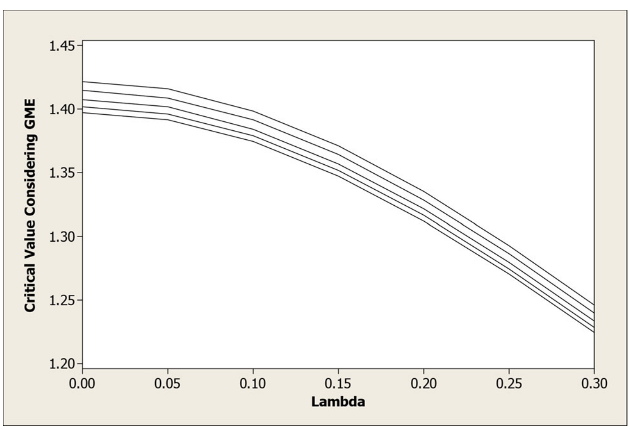

Figure 10. A plot of c_0 with λ for $C_{AQL} = 1.55$, $C_{LPTD} = 1.00$, $\alpha = 0.025$, $\beta = 0.010, 0.025, 0.050, 0.075, 0.100$ (from top to bottom).

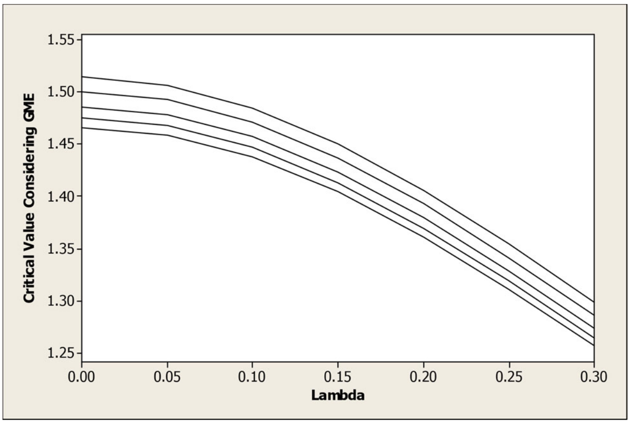

Figure 11. A plot of c_0 with λ for $C_{AQL} = 1.67$, $C_{LPTD} = 1.33$, $\alpha = 0.025$, $\beta = 0.010, 0.025, 0.050, 0.075, 0.100$ (from top to bottom).

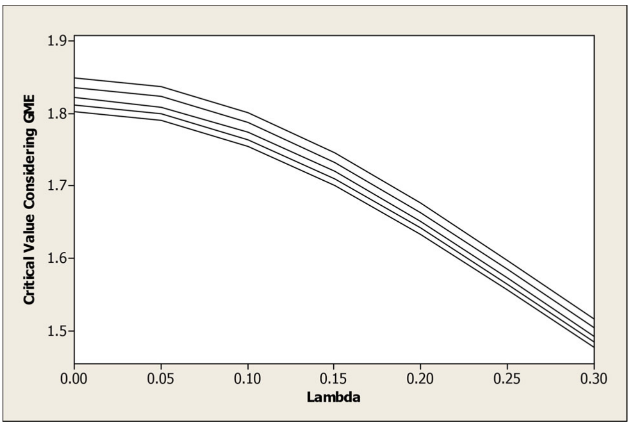

Figure 12. A plot of c_0 with λ for $C_{AQL} = 2.00$, $C_{LPTD} = 1.00$, $\alpha = 0.025$, $\beta = 0.010, 0.025, 0.050, 0.075, 0.100$ (from top to bottom).

Step 1: Determine the process capability requirements (i.e. C_{AQL}, C_{LTPD}), α, β and λ.
Step 2: Check Tables 3–10 to find the required sample size and the corresponding critical acceptance value, (n, c_0) based on the given values of Step 1.
Step 3: Calculate the estimate $\hat{C}_{pk}^{Y}$ from the n sampled data for inspection.
Step 4: Decide to either accept the entire lot if the estimated $\hat{C}_{pk}^{Y}$ value is greater than the critical value c_0, (i.e., $\hat{C}_{pk}^{Y} > c_0$) or to otherwise, reject the entire lot.

3.2. Case study

A bilateral switch, widely used for electronic devices, is designed to conduct or isolate analog or digital signals (both voltage and current) to support analog applications (i.e. audio and video data transmission). One of the main parameters of a bilateral switch is its signal range; voltages values should be within specification limits. If these are below or above the limits, the component will be damaged. A particular bilateral switch is a high-speed Si-gate complementary metal-oxide-semiconductor (CMOS). Its features

Table 4. Required sample sizes (n) and critical values (c_0) for various α and β with selected $C_{AQL} = 1.33$, $C_{LTPD} = 1.00$, and $\lambda = 0.25, 0.30$.

		λ			
		0.25		0.3	
α	β	n	c_0	n	c_0
0.01	0.01	162	1.0753	163	1.0421
	0.025	135	1.0627	137	1.0298
	0.05	115	1.0501	116	1.0176
	0.075	102	1.0408	103	1.0086
	0.10	93	1.0329	94	1.0009
0.025	0.01	140	1.0887	141	1.0550
	0.025	115	1.0762	116	1.0429
	0.05	96	1.0634	97	1.0306
	0.075	85	1.0539	86	1.0213
	0.10	77	1.0459	77	1.0135
0.05	0.01	122	1.1023	123	1.0682
	0.025	99	1.0900	100	1.0563
	0.05	82	1.0774	83	1.0441
	0.075	71	1.0679	72	1.0349
	0.10	64	1.0597	64	1.0269
0.075	0.01	111	1.1124	112	1.0780
	0.025	89	1.1005	90	1.0665
	0.05	73	1.0882	74	1.0545
	0.075	63	1.0787	64	1.0454
	0.10	56	1.0706	56	1.0375
0.10	0.01	103	1.1210	104	1.0863
	0.025	82	1.1095	83	1.0752
	0.05	66	1.0975	67	1.0635
	0.075	57	1.0883	58	1.0546
	0.10	50	1.0802	51	1.0468

Table 5. Required sample sizes (n) and critical values (c_0) for various α and β with selected $C_{AQL} = 1.50$, $C_{LTPD} = 1.33$ and $\lambda = 0.00, 0.05, 0.10, 0.15, 0.20$.

		λ									
		0.00		0.05		0.10		0.15		0.20	
α	β	n	c_0	n	c_0	n	c_0	n	c_0	n	c_0
0.01	0.01	775	1.4148	775	1.4089	775	1.3916	840	1.3642	845	1.3284
	0.025	704	1.4077	704	1.4018	705	1.3847	708	1.3573	712	1.3217
	0.05	600	1.4005	601	1.3947	602	1.3775	605	1.3504	608	1.3149
	0.075	537	1.3952	538	1.3894	539	1.3723	541	1.3452	545	1.3099
	0.10	491	1.3907	492	1.3849	493	1.3679	495	1.3409	498	1.3057
0.025	0.01	713	1.4222	713	1.4163	715	1.3989	718	1.3713	722	1.3353
	0.025	593	1.4151	593	1.4092	595	1.3919	597	1.3644	600	1.3286
	0.05	498	1.4078	498	1.4019	500	1.3847	502	1.3574	505	1.3218
	0.075	441	1.4024	441	1.3965	442	1.3794	444	1.3522	447	1.3166
	0.10	399	1.3977	400	1.3919	401	1.3748	402	1.3477	405	1.3123
0.05	0.01	616	1.4297	616	1.4237	618	1.4062	621	1.3785	624	1.3423
	0.025	505	1.4227	505	1.4167	506	1.3993	508	1.3717	511	1.3357
	0.05	418	1.4154	418	1.4095	419	1.3922	421	1.3647	423	1.3289
	0.075	366	1.4099	366	1.4040	367	1.3868	368	1.3594	371	1.3237
	0.10	328	1.4052	328	1.3993	329	1.3821	330	1.3549	332	1.3193
0.075	0.01	557	1.4352	557	1.4292	559	1.4117	561	1.3838	564	1.3475
	0.025	451	1.4284	452	1.4224	453	1.4050	455	1.3772	457	1.3411
	0.05	369	1.4212	370	1.4153	371	1.3979	372	1.3704	374	1.3344
	0.075	321	1.4158	321	1.4098	322	1.3925	323	1.3651	325	1.3292
	0.10	285	1.4110	285	1.4051	286	1.3879	287	1.3605	289	1.3247
0.10	0.01	513	1.4399	514	1.4339	515	1.4163	517	1.3884	520	1.3519
	0.025	412	1.4333	413	1.4273	414	1.4098	415	1.3820	418	1.3457
	0.05	334	1.4263	334	1.4203	335	1.4029	337	1.3752	339	1.3391
	0.075	288	1.4209	288	1.4149	289	1.3976	290	1.3700	292	1.3340
	0.10	254	1.4161	255	1.4102	255	1.3929	256	1.3654	258	1.3296

Table 6. Required sample sizes (n) and critical values (c_0) for various α and β with selected $C_{AQL} = 1.50$, $C_{LTPD} = 1.33$ and $\lambda = 0.25, 0.30$.

		λ			
		0.25		0.3	
α	β	n	c_0	n	c_0
0.01	0.01	851	1.2862	859	1.2397
	0.025	718	1.2797	725	1.2335
	0.05	613	1.2732	618	1.2272
	0.075	549	1.2683	554	1.2225
	0.10	502	1.2642	506	1.2185
0.025	0.01	728	1.2929	734	1.2462
	0.025	605	1.2864	610	1.2399
	0.05	509	1.2798	513	1.2335
	0.075	450	1.2748	454	1.2288
	0.10	408	1.2706	412	1.2247
0.05	0.01	629	1.2997	634	1.2527
	0.025	515	1.2933	520	1.2466
	0.05	427	1.2867	430	1.2402
	0.075	373	1.2817	377	1.2354
	0.10	335	1.2774	338	1.2312
0.075	0.01	568	1.3047	573	1.2576
	0.025	461	1.2985	465	1.2516
	0.05	377	1.2920	380	1.2453
	0.075	327	1.2870	330	1.2405
	0.10	291	1.2827	294	1.2363
0.10	0.01	524	1.3090	529	1.2617
	0.025	421	1.3029	424	1.2558
	0.05	341	1.2966	344	1.2497
	0.075	294	1.2916	296	1.2450
	0.10	260	1.2873	262	1.2408

Table 7. Required sample sizes (n) and critical values (c_0) for various α and β with selected $C_{AQL} = 1.67$, $C_{LTPD} = 1.33$ and $\lambda = 0.00, 0.05, 0.10, 0.15, 0.20$.

		λ									
		0.00		0.05		0.10		0.15		0.20	
α	β	n	c_0	n	c_0	n	c_0	n	c_0	n	c_0
0.01	0.01	232	1.4994	232	1.4920	233	1.4702	234	1.4360	236	1.3919
	0.025	195	1.4853	195	1.4779	196	1.4564	197	1.4225	198	1.3789
	0.05	166	1.4712	166	1.4638	166	1.4425	167	1.4089	168	1.3657
	0.075	148	1.4607	148	1.4534	148	1.4323	149	1.3989	150	1.3559
	0.10	135	1.4519	135	1.4447	135	1.4236	136	1.3905	137	1.3478
0.025	0.01	200	1.5144	200	1.5068	201	1.4849	201	1.4503	203	1.4057
	0.025	165	1.5003	166	1.4928	166	1.4711	167	1.4368	168	1.3927
	0.05	139	1.4860	139	1.4786	139	1.4571	140	1.4232	140	1.3794
	0.075	122	1.4754	122	1.4680	123	1.4466	123	1.4129	124	1.3695
	0.10	110	1.4663	111	1.4590	111	1.4377	111	1.4042	112	1.3611
0.05	0.01	174	1.5294	174	1.5218	175	1.4997	175	1.4648	176	1.4197
	0.025	142	1.5157	142	1.5081	142	1.4862	143	1.4516	144	1.4070
	0.05	117	1.5016	117	1.4941	118	1.4723	118	1.4380	119	1.3938
	0.075	102	1.4908	102	1.4834	103	1.4618	103	1.4278	104	1.3839
	0.10	91	1.4816	92	1.4743	92	1.4528	92	1.4189	93	1.3753
0.075	0.01	158	1.5407	158	1.5330	159	1.5107	159	1.4755	160	1.4302
	0.025	128	1.5273	128	1.5197	128	1.4976	129	1.4627	129	1.4178
	0.05	104	1.5134	104	1.5059	105	1.4840	105	1.4494	106	1.4049
	0.075	90	1.5028	90	1.4954	90	1.4736	91	1.4393	91	1.3950
	0.10	80	1.4937	80	1.4862	80	1.4646	81	1.4305	81	1.3865
0.10	0.01	146	1.5502	146	1.5425	147	1.5200	147	1.4847	148	1.4390
	0.025	117	1.5373	117	1.5297	118	1.5074	118	1.4723	119	1.4270
	0.05	95	1.5238	95	1.5162	95	1.4941	95	1.4593	96	1.4145
	0.075	81	1.5134	81	1.5058	82	1.4839	82	1.4493	83	1.4048
	0.10	72	1.5043	72	1.4968	72	1.4750	72	1.4406	73	1.3963

Table 8. Required sample sizes (n) and critical values (c_0) for various α and β with selected $C_{AQL} = 1.67$, $C_{LTPD} = 1.33$ and $\lambda = 0.25, 0.30$.

		λ			
		0.25		0.3	
α	β	n	c_0	n	c_0
0.01	0.01	238	1.3407	240	1.2853
	0.025	199	1.3281	201	1.2732
	0.05	169	1.3154	171	1.2610
	0.075	151	1.3061	153	1.2520
	0.10	138	1.2982	139	1.2445
0.025	0.01	204	1.3541	206	1.2980
	0.025	169	1.3415	171	1.2860
	0.05	142	1.3287	143	1.2737
	0.075	125	1.3192	126	1.2646
	0.10	113	1.3110	114	1.2568
0.05	0.01	178	1.3676	179	1.3110
	0.025	145	1.3553	146	1.2992
	0.05	120	1.3426	121	1.2870
	0.075	104	1.3330	105	1.2778
	0.10	93	1.3247	94	1.2699
0.075	0.01	161	1.3776	163	1.3206
	0.025	130	1.3656	132	1.3091
	0.05	106	1.3532	107	1.2972
	0.075	92	1.3437	93	1.2881
	0.10	82	1.3355	83	1.2802
0.10	0.01	149	1.3861	151	1.3287
	0.025	120	1.3746	121	1.3177
	0.05	97	1.3624	98	1.3060
	0.075	83	1.3531	84	1.2971
	0.10	73	1.3450	74	1.2893

Table 9. Required sample sizes (n) and critical values (c_0) for various α and β with selected $C_{AQL} = 2.00$, $C_{LTPD} = 1.67$ and $\lambda = 0.00, 0.05, 0.10, 0.15, 0.20$.

		λ									
		0.00		0.05		0.10		0.15		0.20	
α	β	n	c_0	n	c_0	n	c_0	n	c_0	n	c_0
0.01	0.01	357	1.8345	358	1.8221	359	1.7865	360	1.7315	362	1.6624
	0.025	301	1.8207	301	1.8085	302	1.7731	303	1.7185	305	1.6499
	0.05	256	1.8069	256	1.7947	257	1.7596	258	1.7055	259	1.6374
	0.075	229	1.7967	229	1.7846	229	1.7497	230	1.6958	232	1.6281
	0.10	209	1.7881	209	1.7760	209	1.7413	210	1.6877	212	1.6203
0.025	0.01	307	1.8489	307	1.8364	308	1.8005	309	1.7451	311	1.6755
	0.025	254	1.8352	255	1.8228	255	1.7872	256	1.7322	258	1.6630
	0.05	213	1.8212	213	1.8090	214	1.7736	215	1.7190	216	1.6504
	0.075	188	1.8108	189	1.7986	189	1.7634	190	1.7091	191	1.6409
	0.10	170	1.8019	171	1.7898	171	1.7548	172	1.7007	173	1.6328
0.05	0.01	266	1.8635	266	1.8509	267	1.8148	268	1.7589	270	1.6887
	0.025	218	1.8501	218	1.8376	218	1.8017	219	1.7462	221	1.6765
	0.05	180	1.8362	180	1.8238	180	1.7882	181	1.7331	182	1.6639
	0.075	157	1.8257	157	1.8134	158	1.7779	158	1.7232	159	1.6544
	0.10	141	1.8167	141	1.8044	141	1.7691	142	1.7147	143	1.6462
0.075	0.01	241	1.8743	242	1.8617	242	1.8253	243	1.7691	245	1.6985
	0.025	195	1.8613	195	1.8487	196	1.8126	197	1.7568	198	1.6867
	0.05	159	1.8476	160	1.8352	160	1.7993	161	1.7439	162	1.6743
	0.075	138	1.8372	138	1.8248	139	1.7892	139	1.7341	140	1.6649
	0.10	123	1.8282	123	1.8159	123	1.7804	124	1.7255	124	1.6567
0.10	0.01	223	1.8836	223	1.8709	224	1.8343	225	1.7778	226	1.7068
	0.025	179	1.8709	179	1.8583	179	1.8220	180	1.7659	181	1.6954
	0.05	145	1.8576	145	1.8451	145	1.8090	146	1.7533	147	1.6833
	0.075	124	1.8473	125	1.8349	125	1.7990	125	1.7436	126	1.6740
	0.10	110	1.8384	110	1.8260	110	1.7903	111	1.7352	111	1.6659

Table 10. Required sample sizes (n) and critical values (c_0) for various α and β with selected $C_{AQL} = 2.00$, $C_{LTPD} = 1.67$ and $\lambda = 0.25, 0.30$.

		λ			
		0.25		0.3	
α	β	n	c_0	n	c_0
0.01	0.01	365	1.5846	368	1.5029
	0.025	307	1.5727	310	1.4916
	0.05	261	1.5608	264	1.4803
	0.075	233	1.5520	236	1.4719
	0.10	213	1.5445	215	1.4649
0.025	0.01	313	1.5971	316	1.5147
	0.025	260	1.5852	262	1.5035
	0.05	218	1.5732	220	1.4920
	0.075	192	1.5641	194	1.4835
	0.10	174	1.5564	176	1.4762
0.05	0.01	272	1.6097	274	1.5267
	0.025	222	1.5981	224	1.5156
	0.05	184	1.5861	185	1.5043
	0.075	160	1.5770	162	1.4957
	0.10	144	1.5692	145	1.4882
0.075	0.01	246	1.6190	249	1.5355
	0.025	199	1.6077	201	1.5248
	0.05	163	1.5959	164	1.5136
	0.075	141	1.5869	142	1.5051
	0.10	125	1.5791	126	1.4977
0.10	0.01	228	1.6270	230	1.5431
	0.025	182	1.6160	184	1.5327
	0.05	148	1.6045	149	1.5218
	0.075	127	1.5957	128	1.5133
	0.10	112	1.5879	113	1.5060

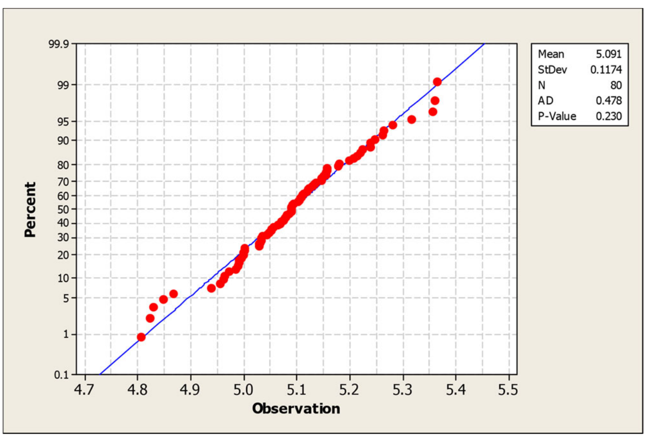

Figure 13. Normal probability plot of data.

include high noise immunity, low power dissipation, and balanced propagation delays. The electrical characteristic of the bilateral switch is Supply Voltage(V_{cc}) and the specification limits are set to USL = 5.5 V, T = 5.0 V, LSL = 4.5 V. This specification can be found in 74V2T66 datasheet, built by STmicroelectronics. The C_{AQL} and C_{LPTD} are set to 1.33 and 1.00 with $\alpha = 0.05$ and $\beta = 0.05$. Based on Table 3, the required sample size and the acceptance critical value are 80 and 1.1669, respectively. Data randomly

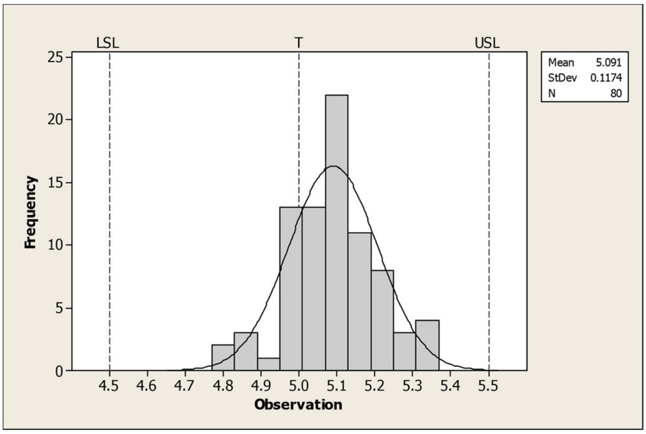

Figure 14. Data histogram.

Table 11. Sample data; 80 observations.

5.264	5.028	4.806	5.090	5.052	5.093	5.110	5.120
5.051	5.089	5.237	4.999	5.315	5.084	5.088	5.043
5.134	5.260	4.830	5.279	5.112	5.118	5.135	4.866
5.123	5.078	4.994	5.355	4.847	5.218	4.991	4.988
5.146	5.363	5.031	5.033	5.199	5.000	5.359	5.104
4.971	5.212	4.954	5.077	5.047	5.056	5.129	4.938
5.074	5.092	5.154	5.145	5.088	5.105	4.990	5.080
4.984	5.101	5.107	5.001	4.962	5.155	5.157	5.206
5.063	5.035	5.178	5.029	5.032	4.999	4.964	5.071
5.157	5.180	4.822	5.150	5.237	5.224	5.246	5.068

generated from a normal distribution with mean $\mu = 5.00$ and standard deviation $\sigma = 0.10$, is displayed in Table 11. Figures 13 and 14 show that the data follows a normal distribution and in control.

Based on the data, we calculate $\bar{y}$, s_Y, and $\hat{C}_{pk}^{Y}$ as follows:

$$\bar{y} = 5.091$$

$$s_Y = 0.1174$$

$$\hat{C}_{pk}^{Y} = \frac{d - |\bar{y} - M|}{3s_Y} = \frac{0.5 - 0.0910}{3(0.1174)} = 1.1613.$$

The consumer will reject the whole lot since the estimate from the inspection is 1.613 which falls short of the critical acceptance value 1.1669. However, not many factories conducting the gauge R&R study for their processes. Thus, in this case, if the producer assumed that their gauge capability is $\lambda = 0.1$, then the consumer might accept the entire lot since the critical acceptance value factoring GMEs is 1.1511, greater than the sample estimator. Comparison of the acceptance/rejection decision of the lot based on the different value of λ is as follows:

From Table 12 we can see that different λ value results in different required sample size and critical value of acceptance which lead to accept or reject the lot. Thus, overlooking GMEs can lead both parties to a wrong decision.

Table 12. The decision of the lot based on $\alpha = 0.05$, $\beta = 0.05$, $C_{AQL} = 1.33$, $C_{LTPD} = 1.00$ and $\lambda = 0.00, 0.05, 0.10, 0.15, 0.20$.

	λ									
	0.00		0.05		0.10		0.15		0.20	
	n	c_0	n	c_0	n	c_0	n	c_0	n	c_0
	80	1.1669	80	1.1629	80	1.1511	81	1.1322	81	1.1072
Decision	Reject		Accept		Accept		Accept		Accept	

4. Summary and discussion

Assessing lot acceptance based on the sampling plan for particular PCIs is widely used in manufacturing. As one of the important instruments in management, PCIs provide quantitative measures of manufacturing capability according to specification limits. On the other hand, an acceptance sampling plan is practical for the assignment of the product's lot. The combination of both tools is useful to confirm that the products meet high-quality standards. A reliable decision rule for product sentencing is provided for both producers and buyers. In this paper, we provided the sampling plan considering GMEs based on the process capability index C_{pk}, one of the popular indices in this field. The acceptance sampling plan factoring GMEs provides buyers and producers with reliable decision rules for product sentencing without underestimating process capability. To make a trustworthy decision, practitioners might use the required sample size and the critical acceptance value accounted for GMEs.

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